Robotics

Chapter 2 : Spatial descriptions and Transformations

Chapter 2

Spatial descriptions

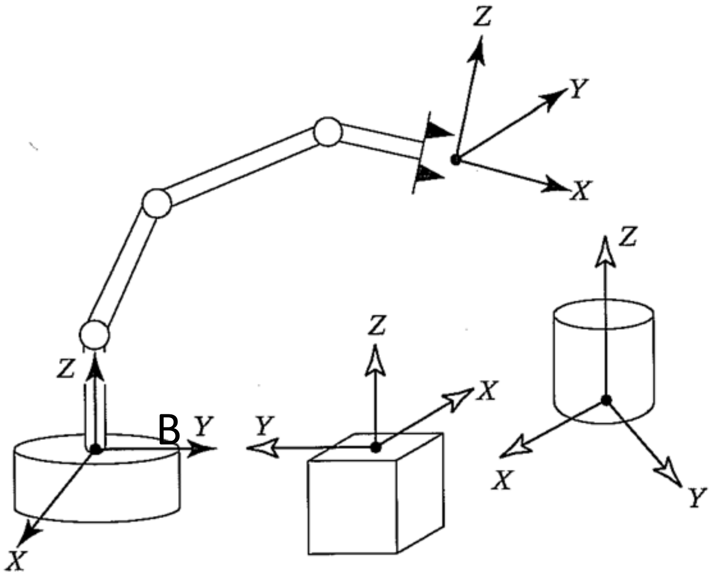
2.2 DESCRIPTIONS : POSITIONS,ORIENTATIONS,AND FRAMES

2.3 MAPPINGS :
CHANGING DESCRIPTION FROM FRAME TO FRAME

2.4 OPERATORS :
TRANSLATIONS ,ROTATIONS, AND TRANSFORMATIONS

Introduction:

We are constantly concerned with the location of objects in three-dimensional space. These objects are the links of the manipulator, the parts and tools with which it deals, and other objects in the manipulator's environment.

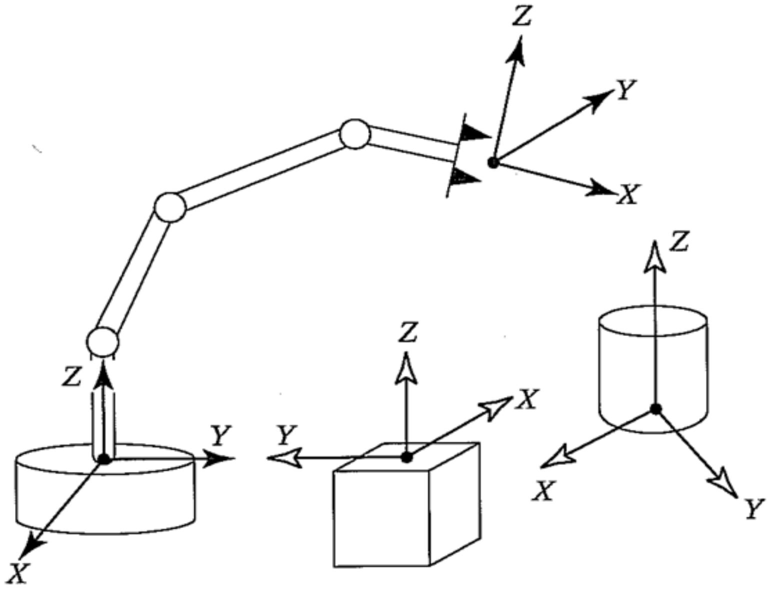


Position and orientation

FIGURE 1.5: Coordinate systems or "frames" are attached to the manipulator and to objects in the environment.

Introduction: cont.

In order to describe the position and orientation of a body in space, we will always attach a coordinate system, or **frame**, rigidly to the object. We then proceed to describe the position and orientation of this frame with respect to some reference coordinate system. (See Fig. 1.5.)

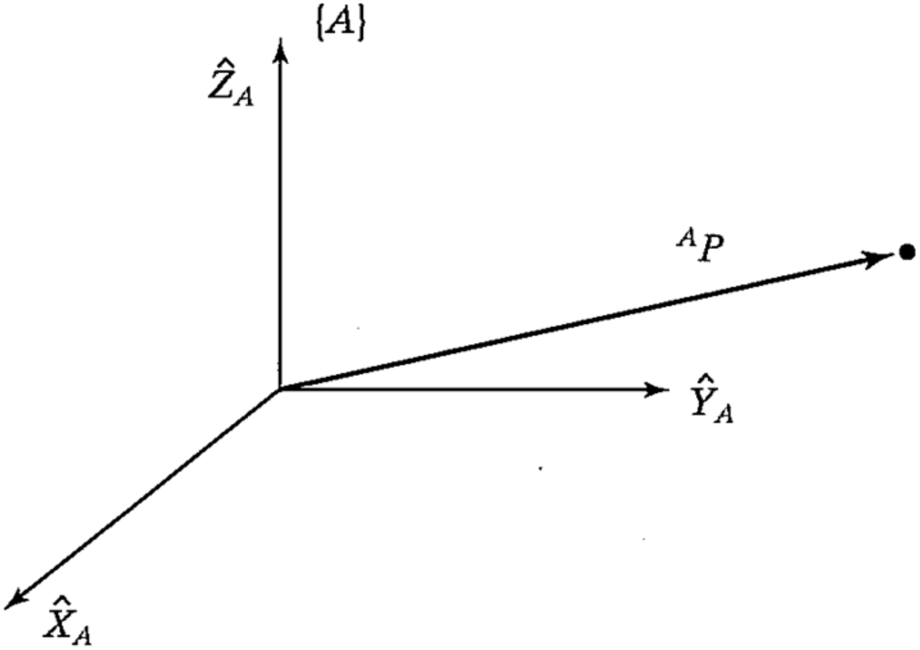


Position and orientation

FIGURE 1.5: Coordinate systems or “frames” are attached to the manipulator and to objects in the environment.

Description of a position

Once a coordinate system is established, we can locate any point in the universe with a 3 × 1 position vector. Because we will often define many coordinate systems, the vector will have the name of the coordinate



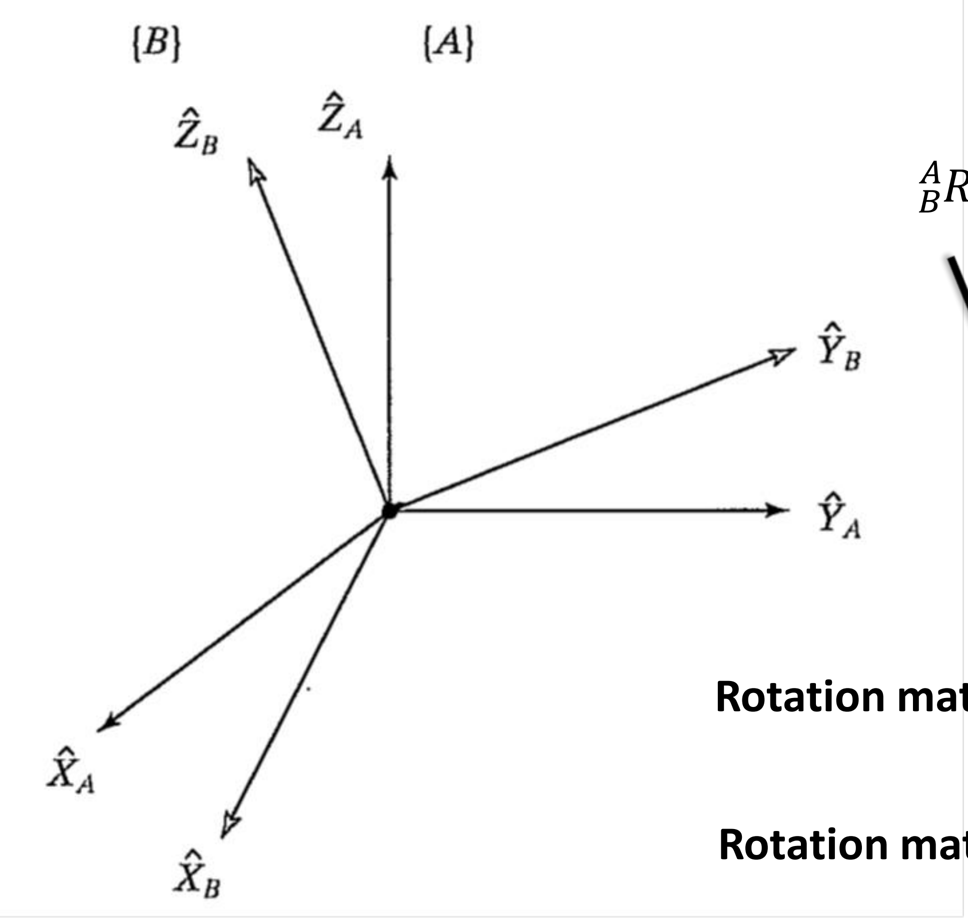
$$|\hat{x}| = 1,$$
$$|\hat{y}| = 1,$$
$$|\hat{z}| = 1,$$

$$A_P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

FIGURE 2.1: Vector relative to frame (example).

Description of an orientation

Often, we will find it necessary not only to represent a point in space but also to describe the **orientation** of a body in space .



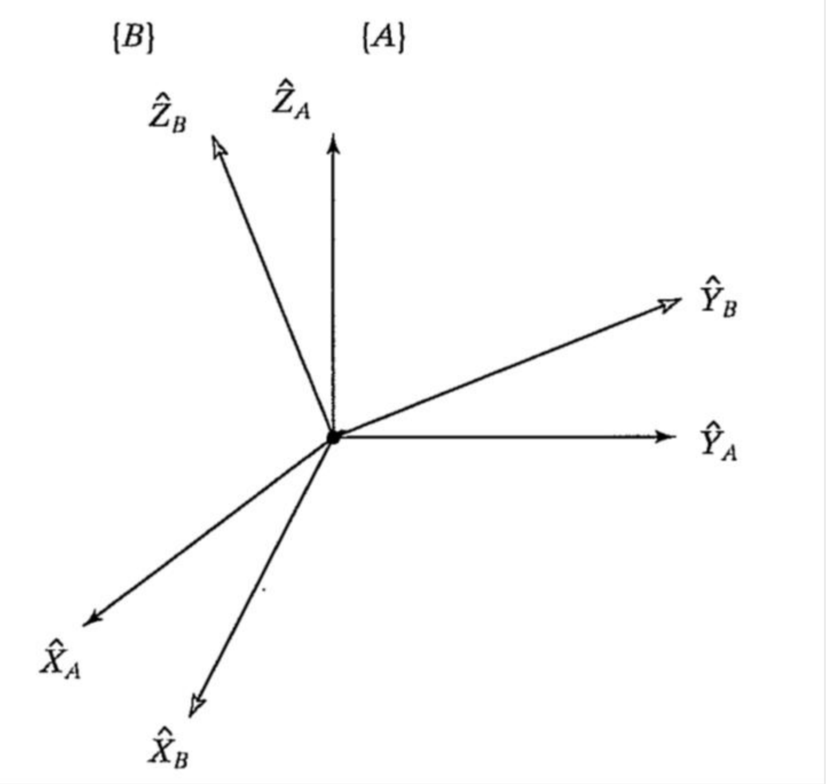
$$ {}^{A}_{B}R = \left[\, \widehat{^{A}X}_B \; \widehat{^{A}Y}_B \; \widehat{^{A}Z}_B \right] = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} $$

Rotation matrix : {B} relative to { A}

Rotation matrix : {B} with respect to (w.r.t) A}

We can give expressions for the scalars r_{ij} in (2.2) by nothing that the components of any vector are simply the projections of that vector on to the unit directions of its reference frame. Hence , each component of ${}^{A}_{B}R$ in (2.2) can be written as the dot product of a pair of unit vectors :

$$ {}^{A}_{B}R = \left[\widehat{{}^{A}X}_B \quad \widehat{{}^{A}Y}_B \quad \widehat{{}^{A}Z}_B \right] = \begin{bmatrix} \hat{X}_B \,.\, \hat{X}_A & \hat{Y}_B \,.\, \hat{X}_A & \hat{Z}_B \,.\, \hat{X}_A \\ \hat{X}_B \,.\, \hat{Y}_A & \hat{Y}_B \,.\, \hat{Y}_A & \hat{Z}_B \,.\, \hat{Y}_A \\ \hat{X}_B \,.\, \hat{Z}_A & \hat{Y}_B \,.\, \hat{Z}_A & \hat{Z}_B \,.\, \hat{Z}_A \end{bmatrix} \quad (2.3) $$



<u>Note: dot product for vector</u>

Find ${}^{B}_{A}R$ given ${}^{A}_{B}R$?

$$ {}^{A}_{B}R = \left[{}^{A}\hat{X}_{B} \quad {}^{A}\hat{Y}_{B} \quad {}^{A}\hat{Z}_{B} \right] = \begin{bmatrix} {}^{B}\hat{X}_{A}^{T} \\ {}^{B}\hat{Y}_{A}^{T} \\ {}^{B}\hat{Z}_{A}^{T} \end{bmatrix}. \tag{2.4} $$

Hence, ${}^{B}_{A}R$*, the description of frame {A} relative to {B}, is given by the transpose of* ${}^{A}_{B}R$*; that is,*

$$ {}^{B}_{A}R = {}^{A}_{B}R^{T}. $$

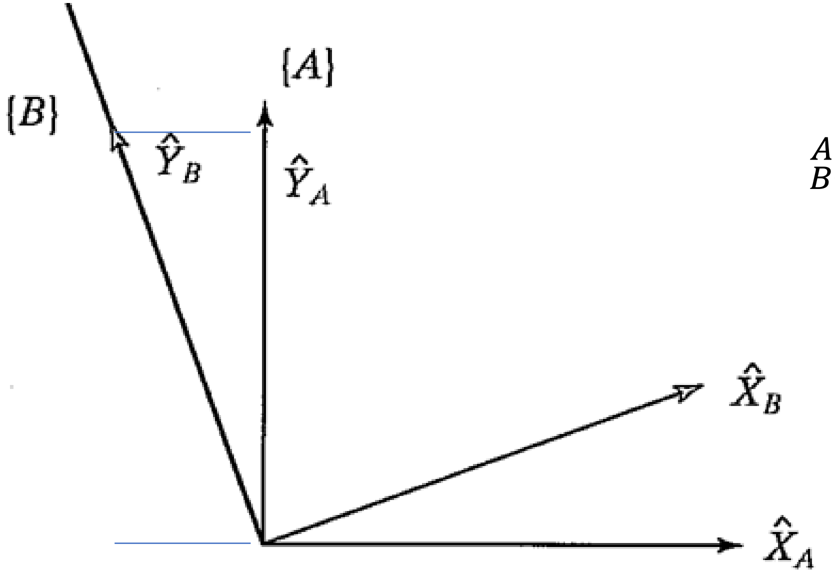
This suggests that the inverse of a rotation matrix is equal to its transpose:

$$ {}^{B}_{A}R = {}^{A}_{B}R^{T} = {}^{A}_{B}R^{-1} $$

$$ {}^{A}_{B}R^{T}\,{}^{A}_{B}R = \begin{bmatrix} {}^{A}\hat{X}_{B}^{T} \\ {}^{A}\hat{Y}_{B}^{T} \\ {}^{A}\hat{Z}_{B}^{T} \end{bmatrix} \left[{}^{A}\hat{X}_{B} \quad {}^{A}\hat{Y}_{B} \quad {}^{A}\hat{Z}_{B} \right] = I_3, $$

EXAMPLE 2.1

Figure 2.6 shows a frame {B} that is rotated relative to frame {A} about $\hat{z}$ by θ =30 degrees. Here , $\hat{z}$ is pointing out of the page . Find the rotation matrix {B} w.r.t {A}



$${}^{A}_{B}R = R_Z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}_{B}R = \begin{bmatrix} 0.866 & -0.500 & 0.000 \\ 0.500 & 0.866 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$$

$${}^{B}_{A}R = {}^{A}_{B}R^T = \begin{bmatrix} 0.866 & 0.500 & 0.000 \\ -0.500 & 0.866 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$$

FIGURE 2.6: {B} rotated 30 degrees about $\hat{z}$.

APPENDIX A

Formulas for rotation about the principle axes by :

$$R_X(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix},$$

$$R_Y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix},$$

$$R_Z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Mappings involving general frames

Very often, we know the description of a vector with respect to some frame {B},and we would like to know its description with respect to another frame,{A}.

We now consider the general case of mapping. Here, the origin of frame {B} is not coincident with that of frame {A} but has a general vector offset. The vector that locates {B}'s origin is called $^{A}P_{BORG}$. Also {B} is rotated with respect to {A}, as described by $^{A}_{B}R$.

Given ^{B}P , we wish to compute ^{A}P , as in Fig. 2.7.

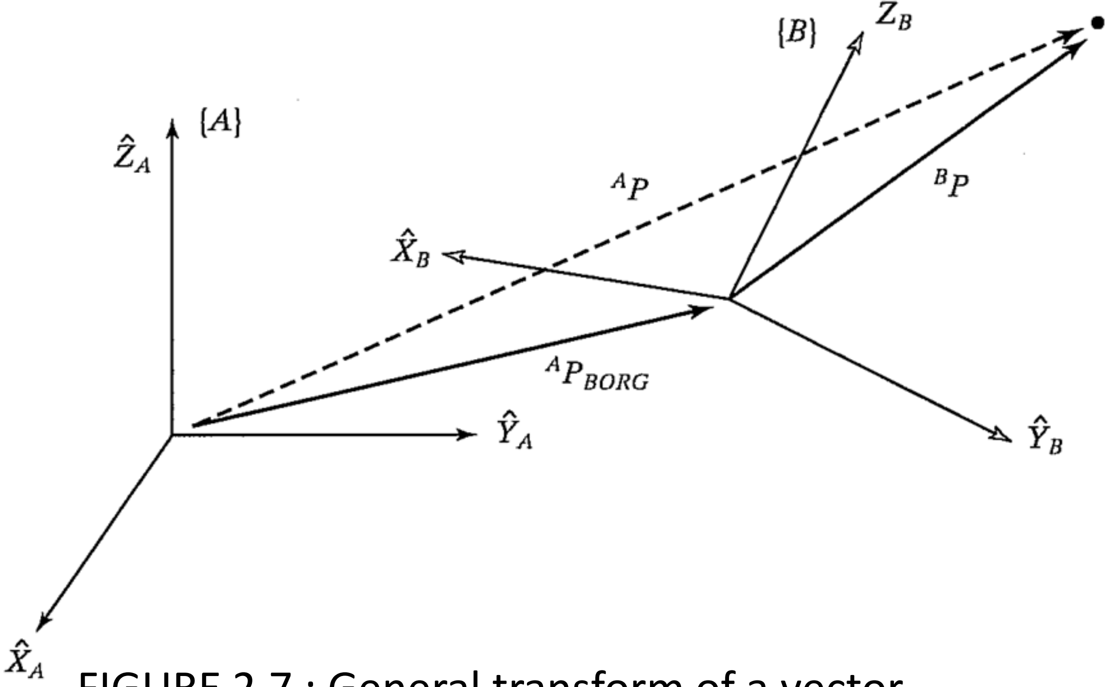
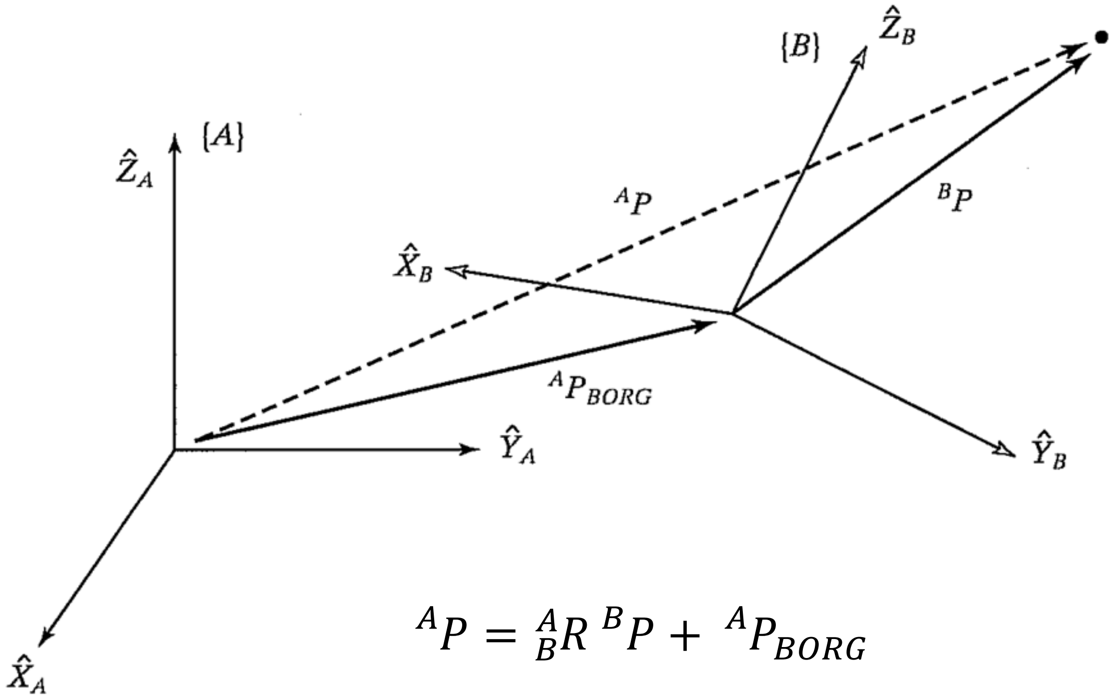


FIGURE 2.7 : General transform of a vector .



$$^{A}P = {}^{A}_{B}R \, {}^{B}P + {}^{A}P_{BORG}$$

Homogeneous transform

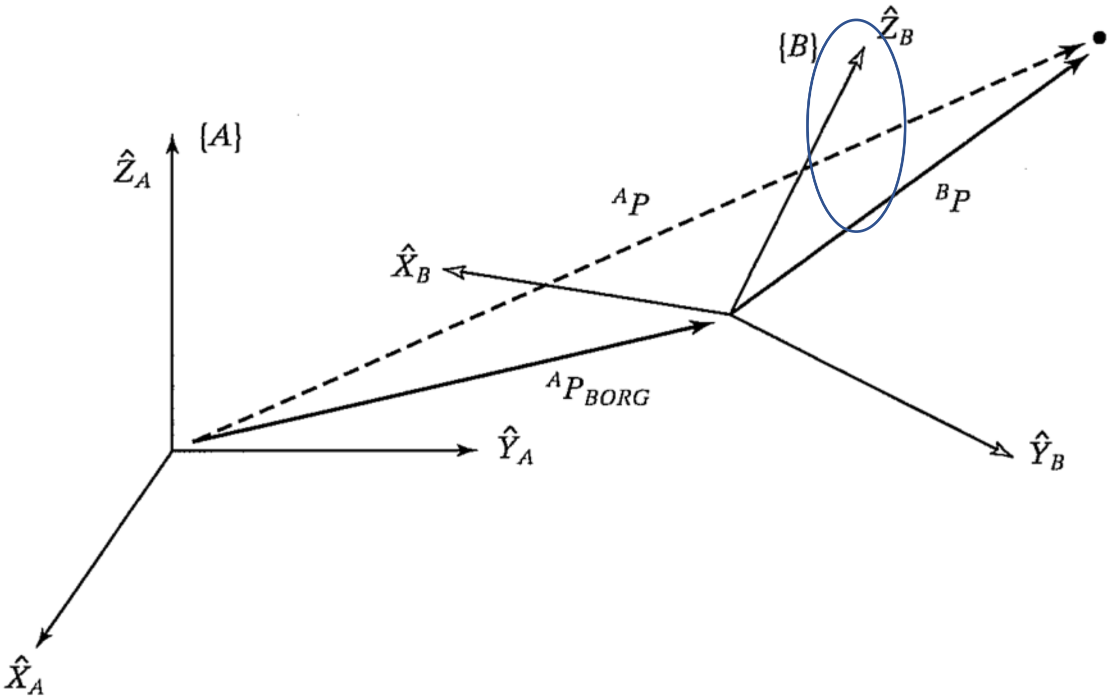
$$^{A}P = {}^{A}_{B}T \, {}^{B}P$$

$$\begin{bmatrix} ^{A}P \\ 1 \end{bmatrix} = \begin{bmatrix} ^{A}_{B}R & ^{A}P_{BORG} \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} ^{B}P \\ 1 \end{bmatrix}$$

EXAMPLE 2.2

Figure shows a frame {B}, which is rotated relative to frame {A} about $\widehat{Z}$ by 30 degree, and translated 10 units in $\hat{X}_A$, and translated 5 units in $\hat{Y}_A$. Find ${}^{A}P$, where ${}^{B}P = [3.0\ 7.0\ 0.0]^T$.

$$ {}^{A}_{B}R = R_Z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.866 & -0.500 & 0.000 \\ 0.500 & 0.866 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix} $$



EXAMPLE 2.5

Figure 2.13 shows a frame {B} that is rotated relative to frame {A} about Z by 30 degrees and translated four units in $\hat{X}_A$ and three units in $\hat{Y}_A$. Thus, we have a description of ${}^{A}_{B}T$.Find ${}^{B}_{A}T$?

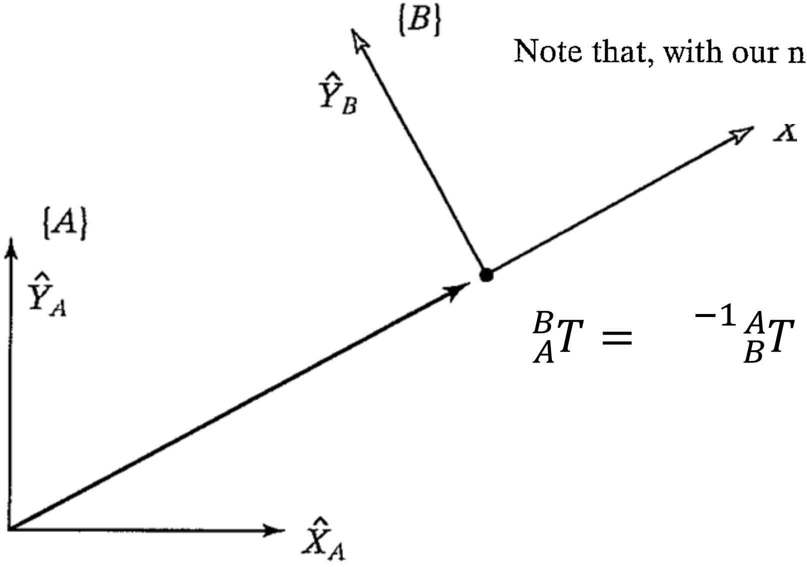
$$
{}^{A}_{B}T = \begin{bmatrix} 0.866 & -0.5 & 0 & 4 \\ 0.5 & 0.866 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2.68)
$$

Using (2.42) and (2.44), we can write the form of ${}^{B}_{A}T$ as

$$
{}^{B}_{A}T = \left[\begin{array}{ccc|c} & {}^{A}_{B}R^{T} & & -{}^{A}_{B}R^{T}\,{}^{A}P_{BORG} \\ \hline 0 & 0 & 0 & 1 \end{array} \right].
$$

Note that, with our notation,

$$
{}^{B}_{A}T = {}^{A}_{B}T^{-1}.
$$

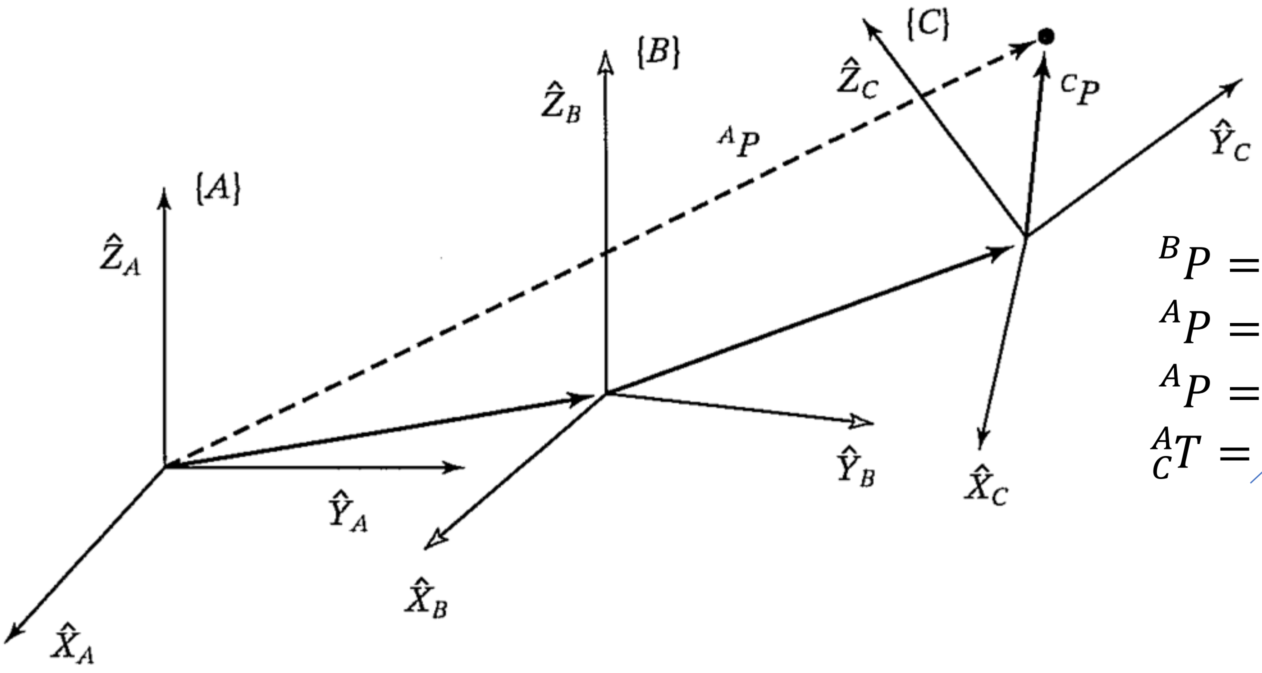


$$
{}^{B}_{A}T = \quad {}^{-1}\,{}^{A}_{B}T = \begin{bmatrix} 0.866 & 0.500 & 0 & -4.964 \\ -0.5 & 0.866 & 0 & -0.598 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

FIGURE 2.13 : {B} relative to {A} .

2.6 Compound transformations

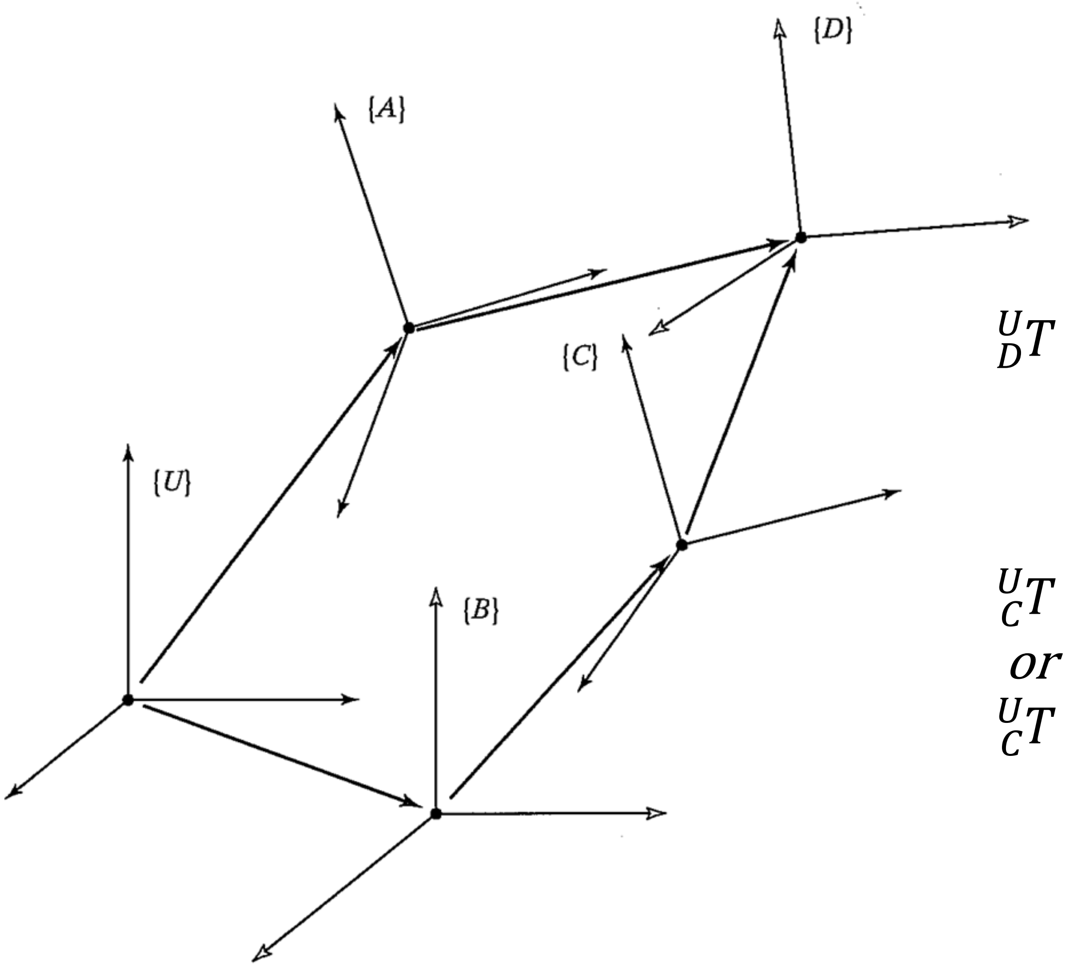
we have ${}^{C}P$ and wish to find ${}^{A}P$.



$$
\begin{aligned}
{}^{B}P &= {}^{B}_{C}T\,{}^{C}P \\
{}^{A}P &= {}^{A}_{B}T\,{}^{B}P \\
{}^{A}P &= {}^{A}_{B}T\,{}^{B}_{C}T\,{}^{C}P \\
{}^{A}_{C}T &= {}^{A}_{\cancel{B}}T\,{}^{\cancel{B}}_{C}T.
\end{aligned}
$$

FIGURE 2.12 : Compound frames: Each is known relative to previous one .

2.7 TRANSFORM EQUATIONS



$$ {}^{U}_{D}T = {}^{U}_{A}T\,{}^{A}_{D}T, or = {}^{U}_{\cancel{B}}T\,{}^{\cancel{B}}_{\cancel{C}}T\,{}^{\cancel{C}}_{D}T. $$

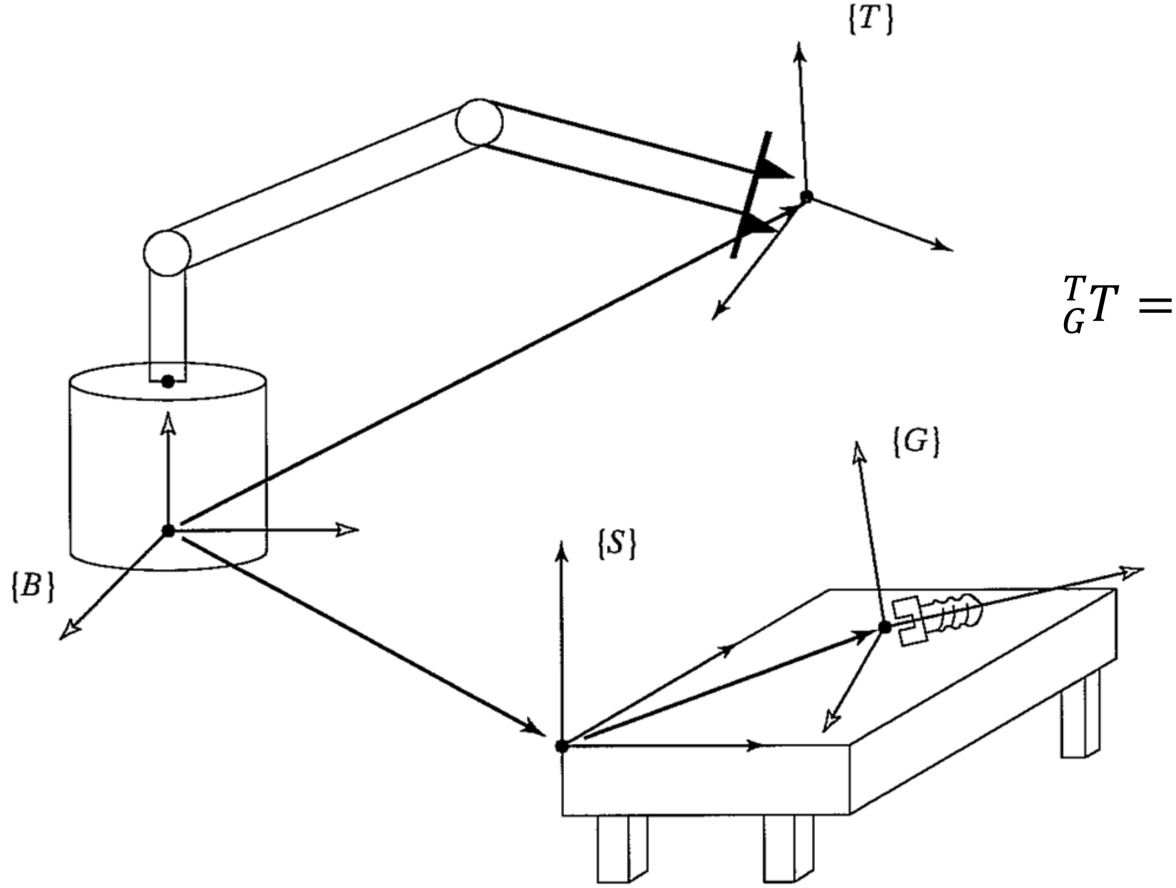
$$ {}^{U}_{C}T = {}^{U}_{A}T\,{}^{A}_{D}T\,{}^{D}_{C}T = {}^{U}_{A}T\,{}^{A}_{D}T\,{}^{C}_{D}T^{-1} $$

or

$$ {}^{U}_{C}T = {}^{U}_{B}T\,{}^{B}_{C}T. $$

FIGURE 2.14: Set of transforms forming a loop .

2.7 TRANSFORM EQUATIONS



$$ {}^{T}_{G}T = {}^{T}_{B}T\,{}^{B}_{S}T\,{}^{S}_{G}T = {}^{B}_{T}T^{-1}\,{}^{B}_{S}T\,{}^{S}_{G}T. $$

FIGURE 2.16 : Manipulator reaching for a bolt .

2.8 MORE ON REPRESENTATION OF ORIANTATION

Rotation matrix determinant is +1

$$R = \left[\hat{X}\ \hat{Y}\ \hat{Z}\right] = \begin{bmatrix} r_{11}\ r_{12}\ r_{13} \\ r_{21}\ r_{22}\ r_{23} \\ r_{31}\ r_{32}\ r_{33} \end{bmatrix}$$

$$\begin{aligned} &\left|\hat{X}\right| = 1, \\ &\left|\hat{Y}\right| = 1, \\ &\left|\hat{Z}\right| = 1, \\ &\hat{X}\, .\hat{Y} = 0, \\ &\hat{X}\ .\ \hat{\mathrm{Z}} = 0, \\ &\hat{Y}\ .\ \hat{Z} = 0. \end{aligned}$$

- Clearly, the nine elements of a rotation matrix are not all independent .
- In fact, given a rotation matrix, R , it is easy to write down the six dependencies between the elements.
- Therefore, rotation matrix can be specified by just three parameters.

2.8 MORE ON REPRESENTATION OF ORIANTATION

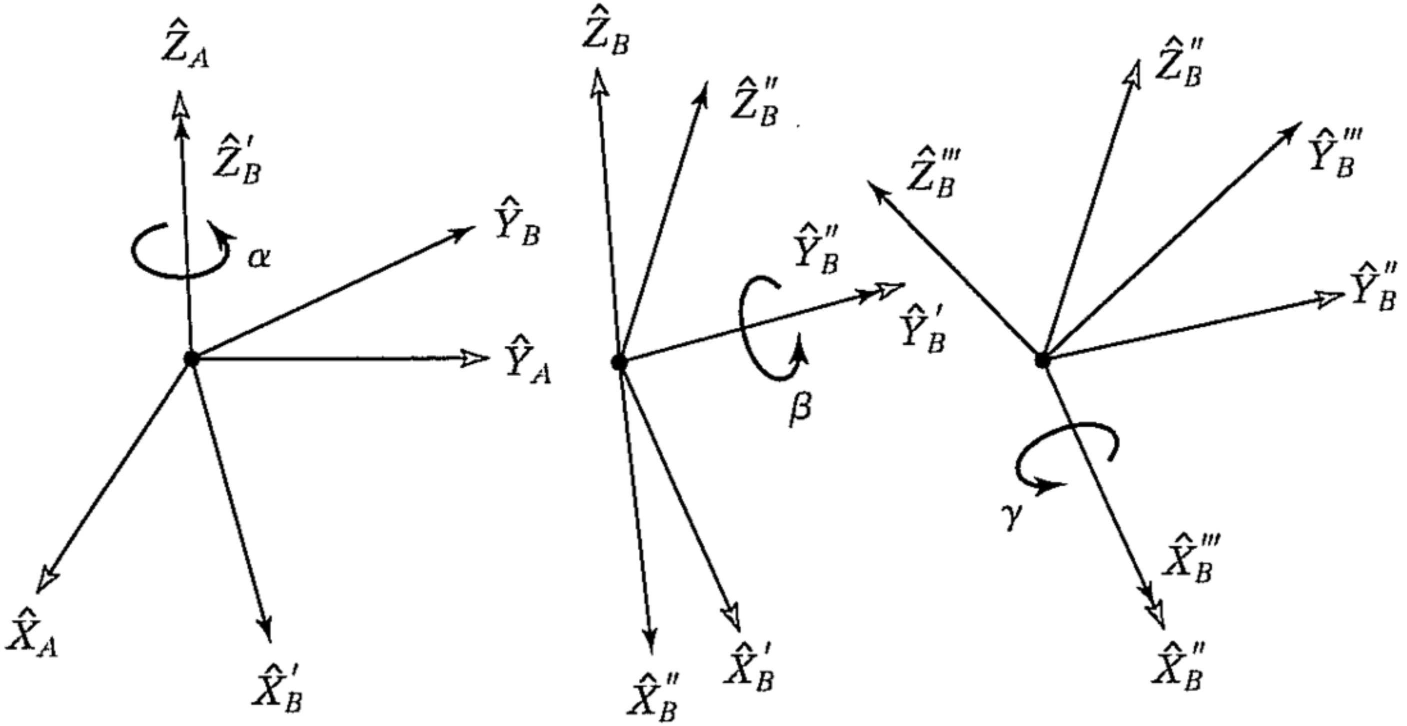
❑ Rotation matrices are useful as operators (computer). Their matrix form is such that, when multiplied by a vector , they perform the rotation operation.

❑ Human operator at a computer terminal who wishes to type in the specification of the desired orientation of a robot’s hand would have a hard time inputting a nine-element matrix with orthonormal columns. A representation that requires only three numbers would be simpler .

Z-Y-X Euler angles (current angles)

Another possible description of a frame {B} is as follows:
Start with the frame coincident with a known frame {A}. Rotate {B} first about $\hat{Z}_B$ by an angle α ,then about $\hat{Y}_B$ by an angle β, and , finally , about $\hat{X}_B$ by an angle γ.

In this representation, each rotation is performed about an axis of the moving system {B} rather than one of the fixed reference {A}. Such sets of three rotations



Z-Y-X Euler angles

$${}^{A}_{B}\boldsymbol{R}_{\acute{Z}\acute{Y}\acute{X}}(\alpha,\beta,\gamma\,) = R_Z(\alpha), R_Y(\beta)R_X(\gamma)$$

$$= \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix}$$

$$= \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma & -s\alpha c\gamma \; c\alpha s\beta c\gamma & +s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma & -c\alpha c\gamma \; s\alpha s\beta c\gamma & +c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma & \end{bmatrix}$$

$$= \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}.$$

given

$$^{A}_{B}\boldsymbol{R}_{\acute{Z}\acute{Y}\acute{X}}(\alpha,\beta,\gamma\ ) = R_Z(\alpha), R_Y(\beta)R_X(\gamma) = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}.$$

$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}.$$

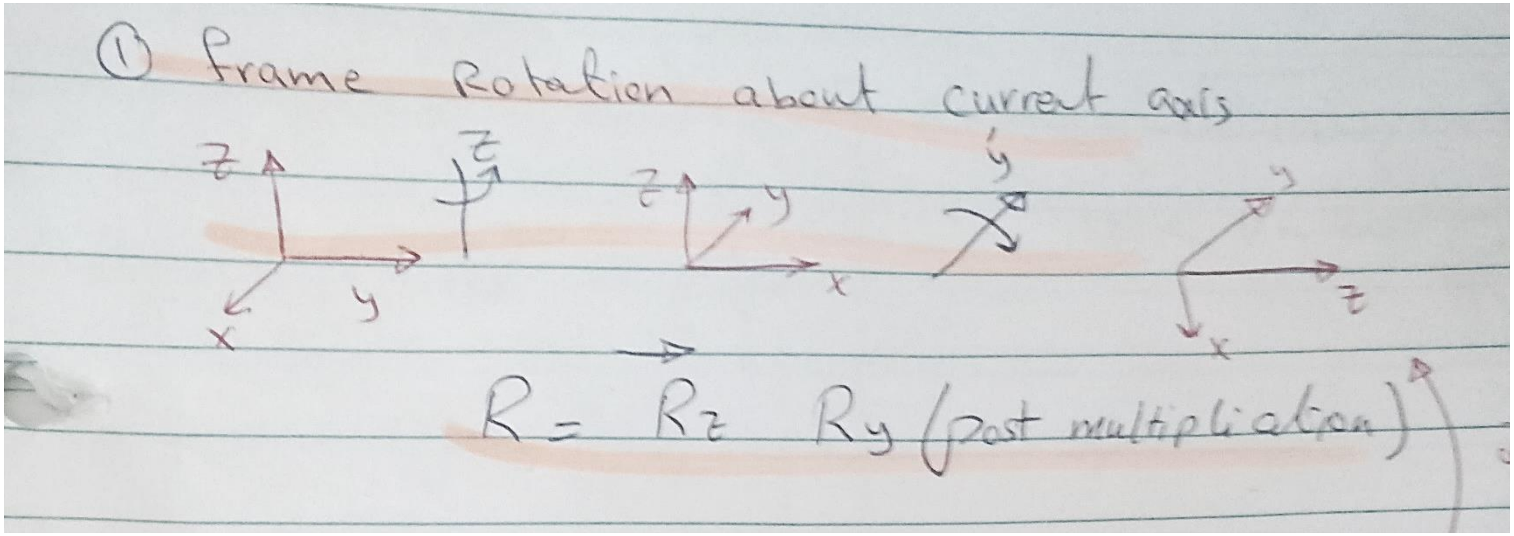
Find

$$\beta = \text{Atan2}(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}),$$

$$\alpha = \text{Atan2}(r_{21}/c\beta, r_{11}/c\beta),$$

$$\gamma = \text{Atan2}(r_{32}/c\beta, r_{33}/c\beta),$$

EXAMPLE



① frame Rotation about current axis

$R = \overrightarrow{R_z}\ R_y$ (post multiplication)

EXAMPLE 2.7

Consider two rotations , one about $\hat{Z}$ by 30 degrees and one about $\hat{X}$ by 30 degrees:

$$R_Z(30) = \begin{bmatrix} 0.866 & -0.500 & 0.000 \\ 0.500 & 0.866 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix} \tag{2.60}$$

$$R_X(30) = \begin{bmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & 0.866 & -0.500 \\ 0.000 & 0.500 & 0.866 \end{bmatrix} \tag{2.61}$$

$$R_Z(30)R_X(30) = \begin{bmatrix} 0.87 & -0.43 & 0.25 \\ 0.50 & 0.75 & -0.43 \\ 0.00 & 0.50 & 0.87 \end{bmatrix}$$

$$\neq R_X(30)R_Z(30) = \begin{bmatrix} 0.87 & -0.50 & 0.00 \\ 0.43 & 0.75 & -0.50 \\ 0.25 & 0.43 & 0.87 \end{bmatrix} \tag{2.62}$$

Z-Y-Z Euler angles

Another possible description of a frame {B} is :
Start with the frame coincident with a known frame {A}.rotate {B} first about $\hat{Z}_B$ by an angle α, then about $\hat{Y}_B$ by an angle β , and , finally , about $\hat{Z}_B$ by an angle γ .

$$ {}^{A}_{B}R_{ZYZ}(\alpha,\beta,\gamma) = \begin{bmatrix} c\alpha c\beta c\gamma - s\alpha s\gamma & -c\alpha c\beta s\gamma - s\alpha c\gamma & c\alpha s\beta \\ s\alpha c\beta c\gamma + c\alpha s\gamma & -s\alpha c\beta s\gamma + c\alpha c\gamma & s\alpha s\beta \\ -s\beta c\gamma & s\beta s\gamma & \mathrm{c}\beta \end{bmatrix}. $$

$$ {}^{A}_{B}R_{ZYZ}(\alpha,\beta,\gamma) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}. $$

$$ \beta = A\tan 2\,(\sqrt{r_{31}^2 + r_{32}^2}, r_{33}), $$
$$ \alpha = A\tan 2\,(r_{23},/s\beta, r_{13}/s\beta), $$
$$ \gamma = A\tan 2\,(r_{32},/s\beta, -r_{31}/s\beta). $$

Given:

$${}^{A}_{B}R_{ZYZ}(\alpha,\beta,\gamma) = \begin{bmatrix} c\alpha c\beta c\gamma - s\alpha s\gamma & -c\alpha c\beta s\gamma - s\alpha c\gamma & c\alpha s\beta \\ s\alpha c\beta c\gamma + c\alpha s\gamma & -s\alpha c\beta s\gamma + c\alpha c\gamma & s\alpha s\beta \\ -s\beta c\gamma & s\beta s\gamma & \mathrm{c}\beta \end{bmatrix}.$$

$${}^{A}_{B}R_{ZYZ}(\alpha,\beta,\gamma) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}.$$

Find $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$

$$\beta = A\tan 2\ (\sqrt{r_{31}^2 + r_{32}^2}, r_{33}),$$
$$\alpha = A\tan 2\ (r_{23},/s\beta, r_{13}/s\beta),$$
$$\gamma = A\tan 2\ (r_{32},/s\beta, -r_{31}/s\beta).$$

$$\beta = \mathrm{Atan2}(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}),$$

$$\alpha = \mathrm{Atan2}(r_{21}/c\beta, r_{11}/c\beta),$$

$$\gamma = \mathrm{Atan2}(r_{32}/c\beta, r_{33}/c\beta),$$

APPENDIX B
The 12 angle-set conventions

The 12 Euler angle sets

2.27 [15] Referring to Fig. 2.25,give the value of ${}^{A}_{B}T$

2.28 [15] Referring to Fig. 2.25,give the value of ${}^{A}_{C}T$

2.29 [15] Referring to Fig. 2.25,give the value of ${}^{B}_{C}T$

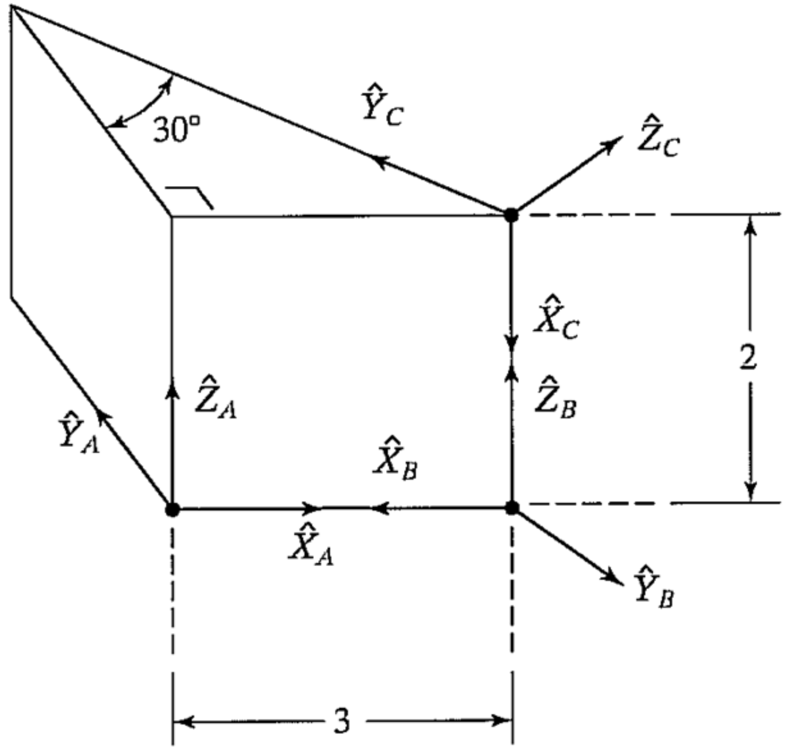
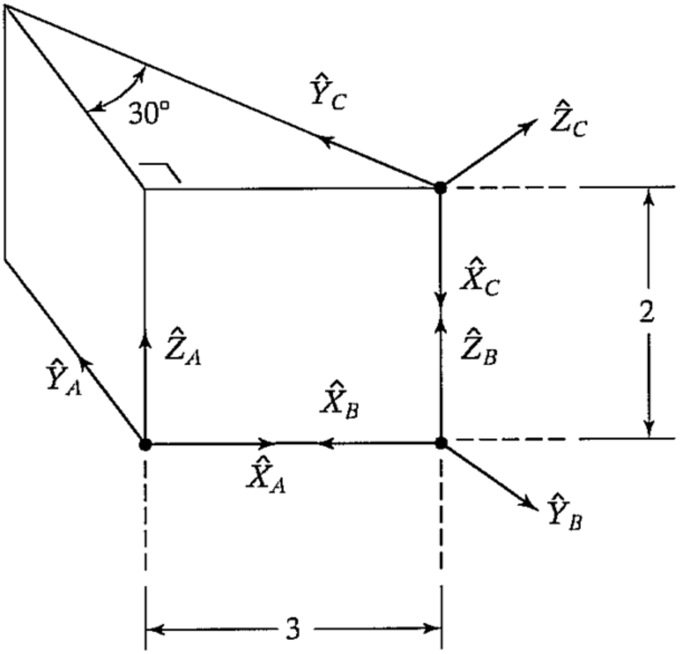
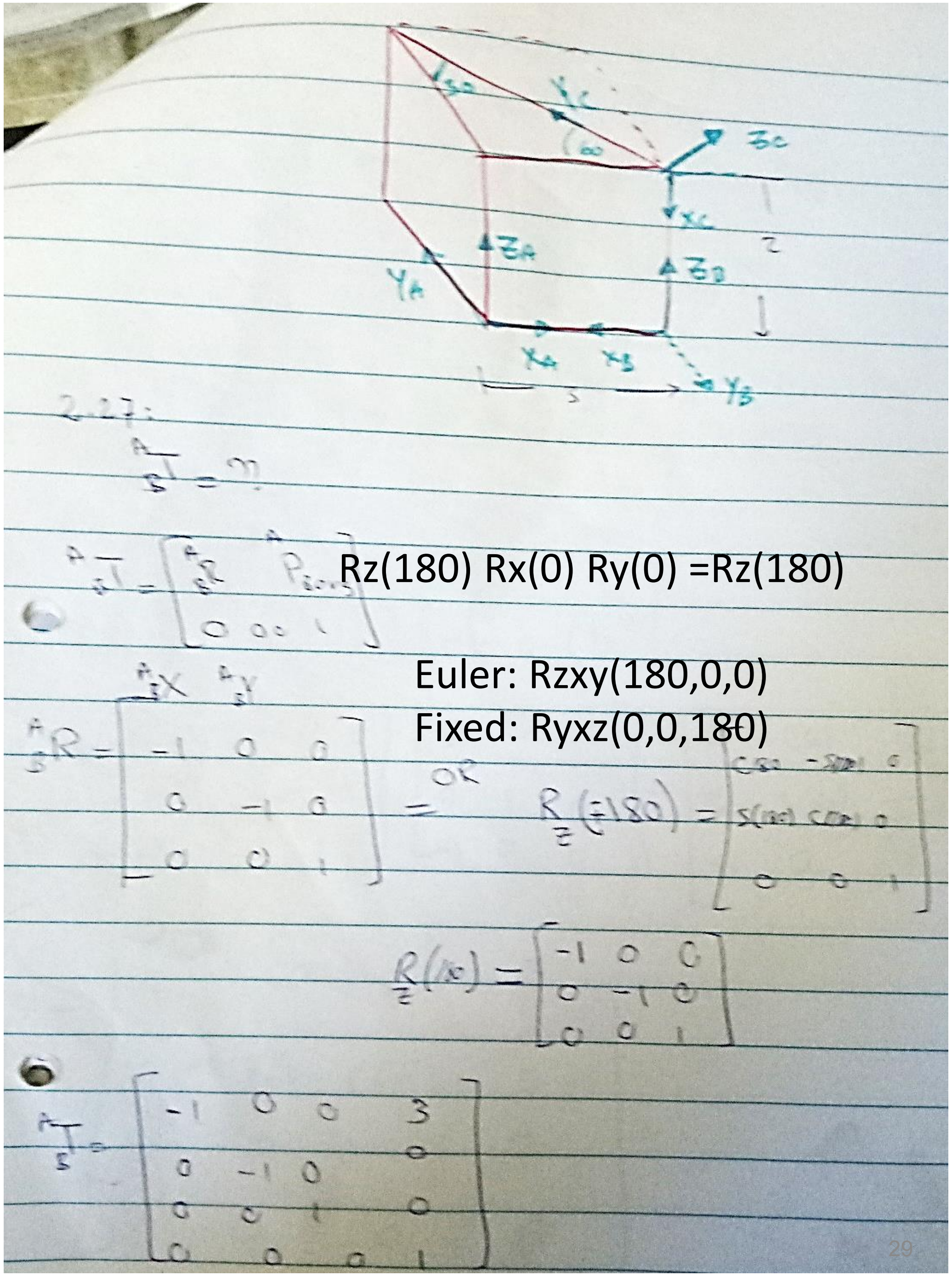


FIGURE 2.25: Frames at the corners of a wedge.

2.27 Fig. 2.25, give the value of ${}^A_BT$





2.27:

${}^A_BT = ?$

$${}^A_BT = \begin{bmatrix} {}^A_BR & {}^AP_{BORG} \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

Rz(180) Rx(0) Ry(0) =Rz(180)

Euler: Rzxy(180,0,0)

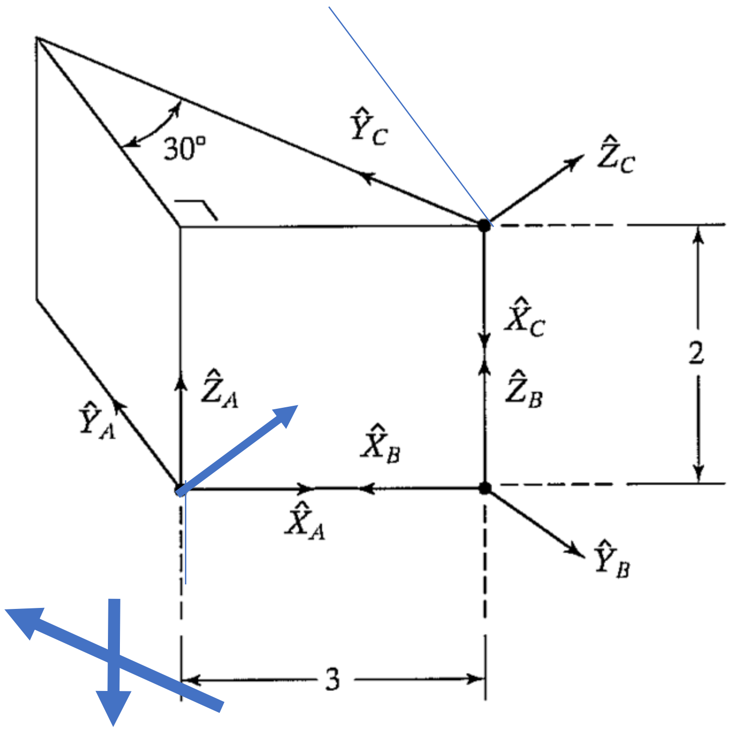
Fixed: Ryxz(0,0,180)

$${}^A_BR = \begin{array}{c} \begin{array}{ccc} {}^A_BX & {}^A_BY & \end{array} \\ \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} = \text{OR} \quad R_Z(\pm 180) = \begin{bmatrix} c180 & -s180 & 0 \\ s(180) & c180 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_Z(180) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^A_BT = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

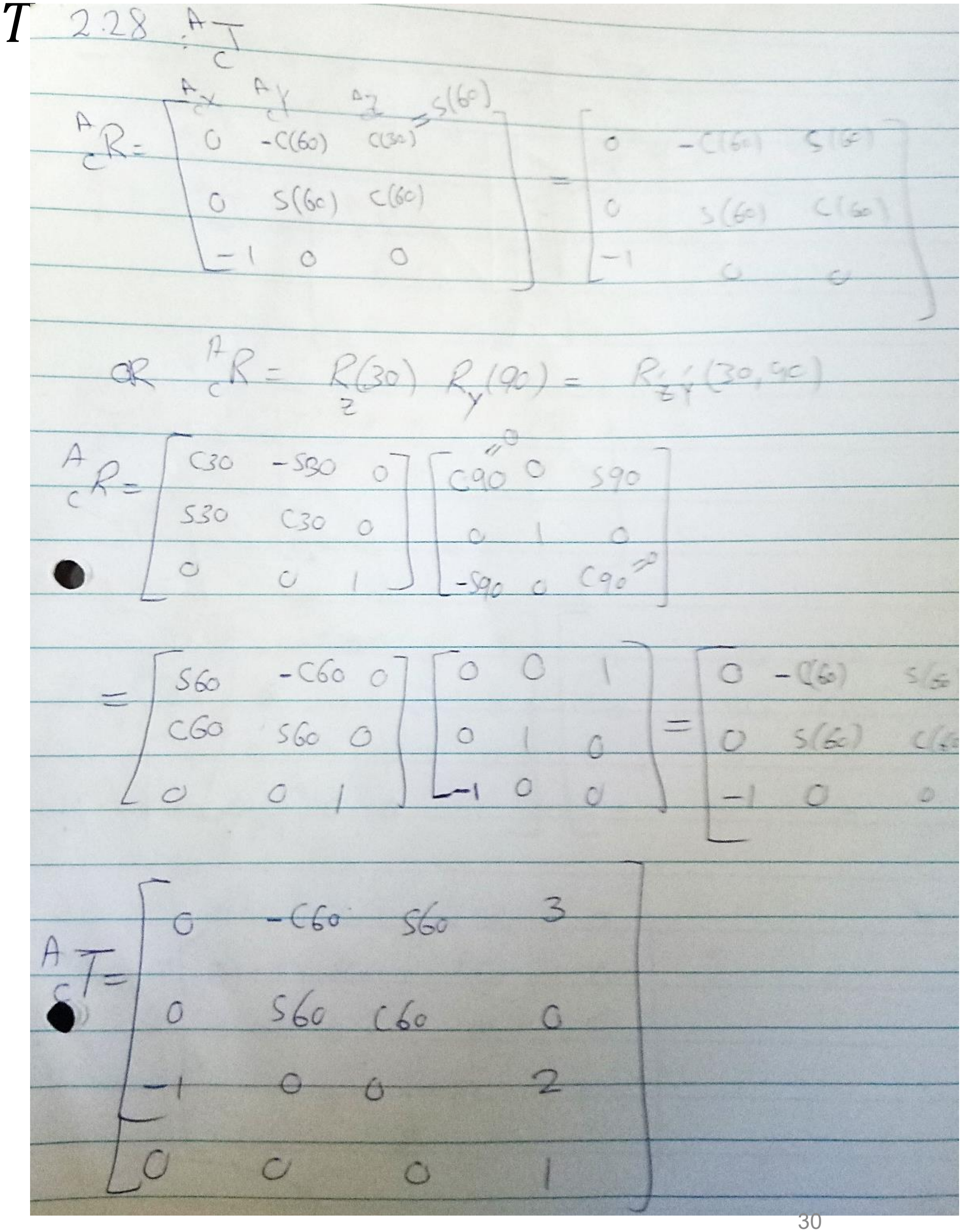
2.28 Fig. 2.25,give the value of ${}^{A}_{C}T$



Ryzx (fixed)=Rx(0) Rz(30) Ry(90)
Rxzy (Euler)=Rx(0) Rz(30) Ry(90)

Other solutions??

Euler: Ry(90) Rx(-30)



2.28 ${}^{A}_{C}T$

$$ {}^{A}_{C}R = \begin{array}{c} \begin{array}{ccc} {}^{A}_{C}x & {}^{A}_{C}Y & {}^{A}_{C}z \end{array} \\ \begin{bmatrix} 0 & -C(60) & C(30) = S(60) \\ 0 & S(60) & C(60) \\ -1 & 0 & 0 \end{bmatrix} \end{array} = \begin{bmatrix} 0 & -C(60) & S(60) \\ 0 & S(60) & C(60) \\ -1 & 0 & 0 \end{bmatrix} $$

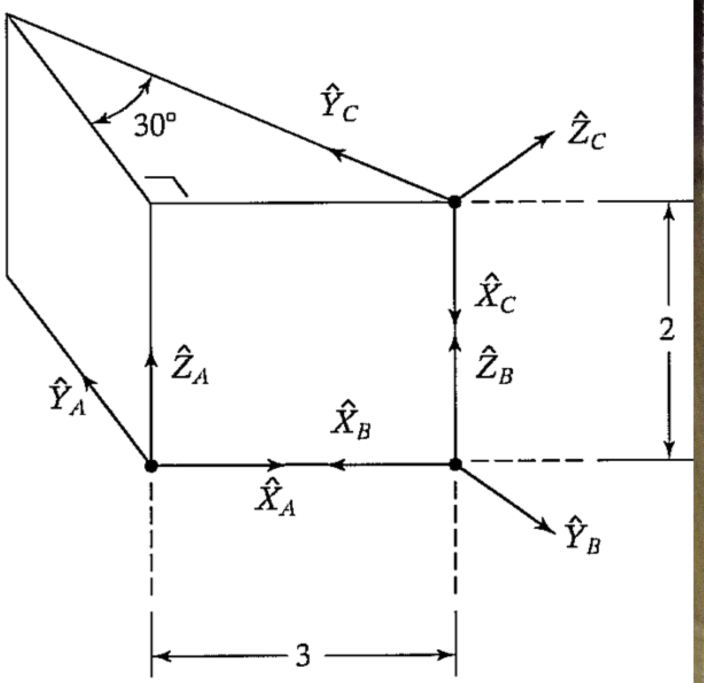
$$ \text{OR} \quad {}^{A}_{C}R = R_z(30)\, R_Y(90) = R_{Z'Y'}(30,90) $$

$$ {}^{A}_{C}R = \begin{bmatrix} C30 & -S30 & 0 \\ S30 & C30 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C90 = 0 & 0 & S90 \\ 0 & 1 & 0 \\ -S90 & 0 & C90 = 0 \end{bmatrix} $$

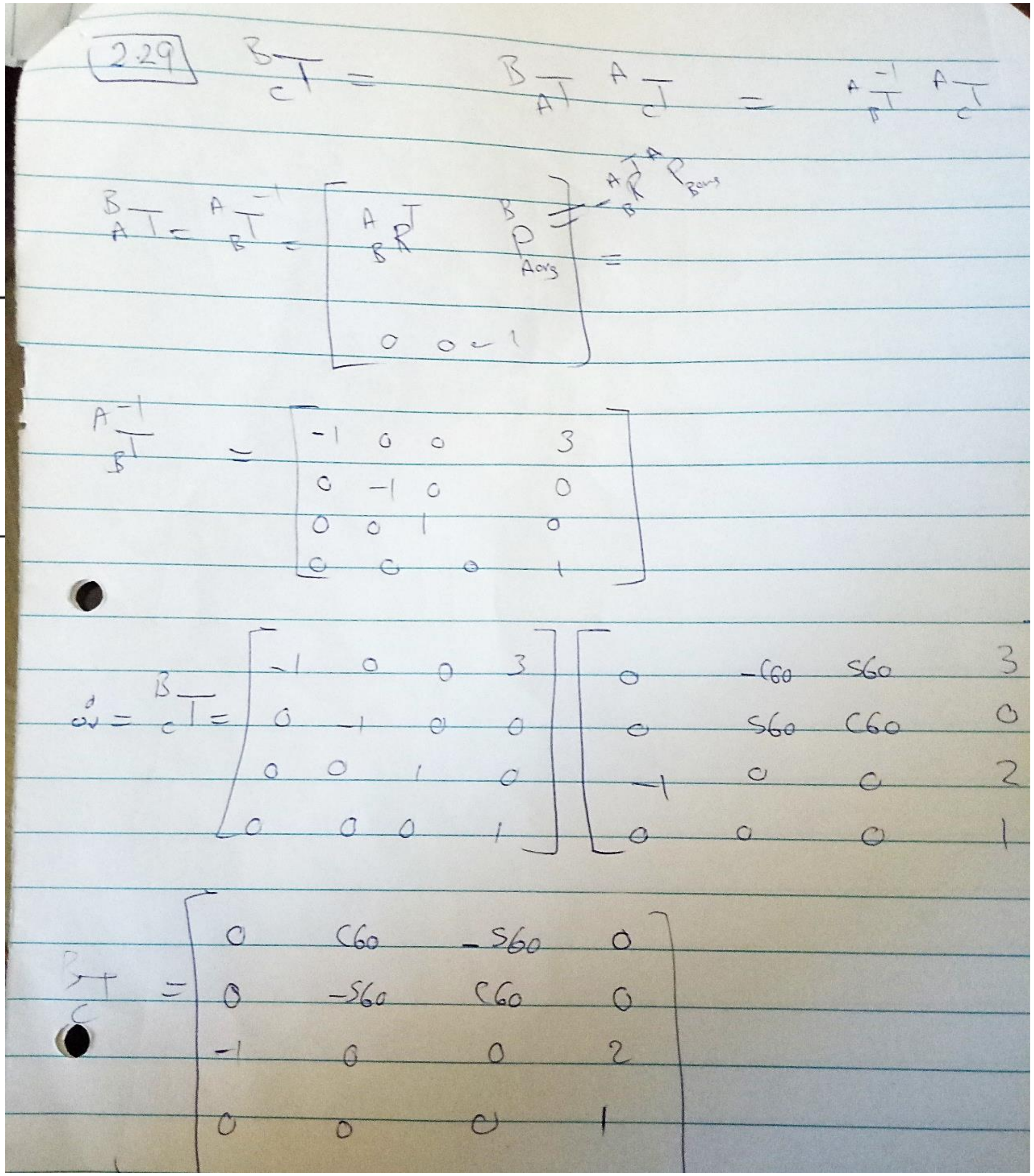
$$ = \begin{bmatrix} S60 & -C60 & 0 \\ C60 & S60 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -C(60) & S(6 \\ 0 & S(60) & C(\\ -1 & 0 & 0 \end{bmatrix} $$

$$ {}^{A}_{C}T = \begin{bmatrix} 0 & -C60 & S60 & 3 \\ 0 & S60 & C60 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

2.29 give the value of ${}^B_C T$



Other solutions??



$\boxed{2.29}$ $\quad {}^B_C T = {}^B_A T \; {}^A_C T = {}^A_B T^{-1} \; {}^A_C T$

$${}^B_A T = {}^A_B T^{-1} = \begin{bmatrix} {}^A_B R^T & {}^B P_{Aorg} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} =$$

$${}^B P_{Aorg} \neq -{}^A_B R^T \; {}^A P_{Borg}$$

$${}^A_B T^{-1} = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[illegible] = {}^B_C T = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -C60 & S60 & 3 \\ 0 & S60 & C60 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^B_C T = \begin{bmatrix} 0 & C60 & -S60 & 0 \\ 0 & -S60 & C60 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$